CORDIC Methods

David Eberly
Geometric Tools, LLC
http://www.geometrictools.com/

Created: October 11, 1999
Last Modified: March 2, 2008

Contents

1 Discussion

CORDIC stands for **Co**ordinate **Ro**tation **Di**gital **C**omputer. The methods were first used in 1959 [1] to solve trigonometric relationships that arose in navigation problems. In the early 1980s, the methods were used by Hewlett-Packard for trigonometric function evaluation on the HP-35 calculator. A good discussion of the topic is [2].

The functions that can be evaluated using CORDIC methods are sine, cosine, tangent, inverse tangent, hyperbolic sine, hyperbolic cosine, hyperbolic tangent, inverse hyperbolic tangent, natural logarithm, natural exponential, square root, multiplication, division. In binary form the scheme consists of the iterative equations

$$\begin{aligned} x_{k+1} &= x_k - m\delta_k y_k 2^{-k} \\ y_{k+1} &= y_k + \delta_k x_k 2^{-k} \\ z_{k+1} &= z_k - \delta_k \epsilon_k \end{aligned}$$

where $m \in \{-1, 0, 1\}$ is a mode indicator, $\{\epsilon_k\}_{k=0}^{n}$ is a sequence of precomputed constants depending on m, and $\delta_k \in \{-1, 1\}$ are appropriately chosen. The initial values x_0, y_0, and z_0 must also be appropriately chosen. The following table provides the necessary values to obtain the aforementioned functions.

	$\delta_k = \left\{ \begin{array}{rl} 1, & z_k \geq 0 \\ -1, & z_k < 0 \end{array} \right\}$	$\delta_k = \left\{ \begin{array}{rl} 1, & y_k < 0 \\ -1, & y_k \geq 0 \end{array} \right\}$
$m = 0$ $\epsilon_k = 2^{-k}$	x_0 given, $y_0 = 0$, z_0 given $y_{n+1} \doteq x_0 z_0$	x_0 given, y_0 given, $z_0 = 0$ $z_{n+1} \doteq y_0/x_0$
$m = 1$ $\epsilon_k = \mathrm{Tan}^{-1}(2^{-k})$ $K = \prod_{j=0}^{n} \cos(\epsilon_j)$	$x_0 = K$, $y_0 = 0$, $z_0 = \theta$ $x_{n+1} \doteq \cos(\theta)$, $y_{n+1} \doteq \sin(\theta)$	x_0 given, y_0 given, $z_0 = 0$ $z_{n+1} \doteq \mathrm{Tan}^{-1}(y_0/x_0)$, $x_{n+1} \doteq K\sqrt{x_0^2 + y_0^2}$
$m = -1$ $\epsilon_k = \mathrm{Tanh}^{-1}(2^{-k})$ $K' = \prod_{j=0}^{n} \cosh(\epsilon_j)$	$x_0 = K'$, $y_0 = 0$, $z_0 = \theta$ $x_{n+1} \doteq \cosh(\theta)$, $y_{n+1} \doteq \sinh(\theta)$	x_0 given, y_0 given, $z_0 = 0$ $z_{n+1} \doteq \mathrm{Tanh}^{-1}(y_0/x_0)$, $x_{n+1} \doteq K'\sqrt{x_0^2 - y_0^2}$ or $x_0 = w + 1$, $y_0 = w - 1$ $z_{n+1} \doteq 0.5 \log w$ or $x_0 = w + 0.25$, $y_0 = w - 0.25$ $x_{n+1} \doteq K'\sqrt{w}$.

Various parameters for the CORDIC scheme.

References

[1] J. Volder, *The CORDIC computing technique*, IRE Trans. Computers, vol. EC-8, pp. 330-334, September 1959.

[2] Charles W. Schelin, *Calculator function approximation*, AMS Monthly, pp. 317-325, May 1983.